PARTIAL DIFFERENTIAL EQUATIONS AND APPLICATIONS

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LECTURES

SOME APPLICATIONS OF A CLOSED-FORM SOLUTION FOR N-FOLD COMPOUND OPTIONS

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A pricing formula for N-fold compound options is derived, solving N nested Black-Scholes differential equations and using multivariate normal integrals. The explicit formula which is proved generalizes Geskes well-known expression. It applies to a wide range of Finance problems, such as the pricing of risky coupon bonds, the valuation of real investments with multiple real options and the analysis of managerial flexibility.

CORES FOR ELLIPTIC DIFFERENTIAL OPERATORS WITH UNBOUNDED COEFFICIENTS ON $\mathbb{R}^N$ AND INVARIANT MEASURES

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Let $A = \sum_{i,j=1}^N a_{ij}(x)D_{ij} + \sum_{i=1}^N b_i(x)D_i$ be an elliptic differential operator with unbounded coefficients on $\mathbb{R}^N$ and assume that the associated Feller semigroup $(T(t))_{t\geq 0}$ has an invariant measure μ. The $(T(t))_{t\geq 0}$ extends to a strongly continuous semigroup $(T_p(t))_{t\geq 0}$ on $L^p(\mu) = L^p(\mathbb{R}^N, \mu)$ for every $1 \leq p < \infty$. We prove that, under mild condition on the coefficients of A, the space of test function $C_c(\mathbb{R}^N)$ is a core for the generator (A_p, D_p) of $(T_p(t))_{t\geq 0}$ i n $L^p(\mu)$ for $1 \leq p < \infty$.

FUNDAMENTAL SOLUTION AND PROPAGATION OF SINGULARITIES IN SOME DEGENERATE HYPERBOLIC CAUCHY PROBLEMS

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We consider second order degenerate hyperbolic equations. The degeneration comes from the weak hyperbolicity or from the too low regularity of the coefficients of the equation with respect to time. We prove C^∞ well posedness of the Cauchy problem for this kind of equations by constructing the fundamental solution for an equivalent first order system and we study the propagation of singularities of the Cauchy data. The fundamental solution is produced as a matrix of Fourier integral operators of order $\delta > 0$, using the method of multi-products of phase functions. Then we study the propagation of singularities of the Cauchy data, showing that if the coefficients of the equation are singular only at $t = 0$, then singularities of the solution propagate along the simple bicharacteristic lines as in the non degenerate case, while if the equation is singular elsewhere we may have a flux of broken bicharacteristics at points of singularity.

ON TIME-PERIODIC SOLUTIONS TO THE NAVIER-STOKES EQUATIONS IN CYLINDRICAL DOMAINS. LERAY'S PROBLEM

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Let A be an open bounded regular set in $\mathbb{R}^n$ and consider the cylinder $B = A \times \mathbb{R}$. Let $g(t)$ be an arbitrary T-periodic real function. We prove that there is one and only one T-periodic solution to the Navier-Stokes equations in B which vanishes on the boundary, is parallel to the axis of the cylinder and, moreover has at any time total flux equal to $g(t)$. The proof follows from a more general theorem. The above result is then applied to extend classical results on the Leray's problem from the stationary to the periodic case.

SURJECTIVE CONVOLUTION OPERATORS ON VECTOR VALUED DISTRIBUTIONS

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We report on joint work with P. Domanski.

The following problem is considered: Suppose that T is a surjective convolution operator on the space of ultradistributions of Beurling type $D'_\omega(G)$ on an open subset G of R^N. Let E be a complete locally convex space. Under which conditions is the vector valued convolution operator $T \otimes Id$ from $D'_\omega(G,E)$ into $D'_\omega(G,E)$ also surjective? Results of this type are motivated by the problem of solving of linear partial differential operators depending on a parameter. The most interesting cases are when E is a Fréchet Schwartz space or a (LS)-space, i.e. the dual of a Fréchet Schwartz space.

Several results were already known. Partial results for operators on spaces of vector valued real analytic functions were obtained by us. In the case of distributions, when T is a hypoelliptic linear partial differential operator $P(D)$ with constant coefficients and G is convex, $P(D) \otimes Id$ is surjective on $D'(G,E)$ for every Fréchet space E and for complete (LB)-spaces E which satisfy the condition (A) of Vogt. These results were obtained by Petszche in 1980. He had continued the work of Vogt. In fact, Vogt proved in 1983 that, for certain elliptic operators $P(D)$, the condition (A) is necessary and sufficient for the surjectivity of $P(D) \otimes Id$ if the space E is a complete (LB)-space.

The problem mentioned above is related to the theory of the vanishing of the functor Ext^1 in the category of (PLS)-spaces (i.e., projective limits of sequences of duals of Fréchet-Schwartz spaces) and with the theory of the vanishing of the first derived functor of the projective limit functor for locally convex spaces. We present characterizations of those pairs (F,X) of a Fréchet Schwartz space F (which is the dual of E in our problem) and an ultrabornological (PLS)-space $X = proj_N X_N$ (which is the kernel of T in our problem) such that every exact sequence of (PLS)-spaces

$$0 \to X \to Y \to F \to 0$$

splits. Our result provides an extension of Vogt Wagner $(DN) - (\Omega)$ splitting theorem. More concrete evaluations for power series spaces F or for a sequence space X, the kernel of a convolution operator on $D'_\omega(G)$ or a space of quasianalytic functions on the real line have been obtained. We are able to omit the ellipticity or hypoellipticity assumptions of Vogt and Petszche, and prove that if

a convolution operator $T : D'_\omega(G) \to D'_\omega(G)$ is surjective (G convex), then the E-valued corresponding operator is surjective, when E is the dual of a power series space of infinite type (for instance, when E is isomorphic to the space of germs of holomorphic functions over a point).

SUBEXPONENTIAL DECAY AND REGULARITY FOR HYPOELLIPTIC OPERATORS

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We discuss the Gevrey regularity and the subexponential decay for solutions of the equation $Pu = f$, where P is a pseudodifferential operator. The symbol of P is assumed to satisfy particular polynomial estimates and identifies a class of symbols which generalizes the classes SG. We study this problem in the functional framework of the Gelfand-Shilov spaces of type S. Starting from a solution u in the dual spaces and from a regular datum f, we are able to precise the regularity and the decay of u, under suitable hypoellipticity assumptions for P. The results reported here have been obtained in collaboration with T. Gramchev and L. Rodino.

ASYMPTOTIC ANALYSIS OF EIGENVALUE PROBLEMS

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We study the asymptotic behaviour of the eigenvalues and eigenfunctions of elliptic problems when the size of the domain on which the problem is set becomes unbounded.

WELL-POSEDNESS FOR A CLASS OF DEGENERATE HYPERBOLIC CAUCHY PROBLEMS

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The aim of this talk is to give an uniform approach to different kinds of degenerate hyperbolic Cauchy problems. We prove that a weakly hyperbolic equation, satisfying an intermediate condition between effective hyperbolicity and the Levi condition, and a strictly hyperbolic equation with non-regular coefficients with respect to the time variable, can be reduced to a first order system of the same type. For such a system we prove an energy estimate in Sobolev spaces (with a loss of derivatives) so obtaining the well-posedness of the Cauchy problem in C^∞.

NONNEGATIVE FUNCTIONS AS SQUARES OR SUMS OF SQUARES

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We prove that, for $n \geq 4$, there are C^∞ nonnegative functions f of n variables (and even flat ones for $n \geq 5$) which are not a finite sum of squares of C^2 functions. For $n = 1$, where a decomposition in a sum of two squares is always possible, we investigate the possibility of writing $f = g^2$. We prove that, in general, one cannot require a better regularity than $g \in C^1$. Assuming that f vanishes at all its local minima, we prove that it is possible to get $g \in C^2$ but that one cannot require any additional regularity. Finally we give a necessary and sufficient condition in order to obtain $g \in C^2$.

These results are contained in a joint paper with J.-M. Bony, F. Broglia and L. Pernazza.

THE OPERATOR SPLITTING METHOD: NONLINEAR HYPERBOLIC BALANCE LAWS AND A GENERALIZATION OF TROTTER-KATO FORMULAS

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The *operator splitting* method has been proved to be a successful technique for solving many evolution problems where the dynamics can be decomposed in two or more simple flows. This is the case, for instance, of nonlinear hyperbolic systems of balance laws, both from the analytical and numerical point of view. We present in this talk some recent results showing two different applications of this method.

First, we consider an $n \times n$ system of hyperbolic balance laws, in one space dimension. The homogeneous part of the system is of Temple type, i.e., shock and rarefaction curves are coinciding. We assume moreover that there exists a set that is invariant both for the conservative part and for the system of ordinary differential equations associated to the source term. By a suitable operator splitting method we prove the well-posedness (existence and continuous dependence from the data) for large times of such a system. Applications to vehicular traffic flow are provided.

Motivated by such results we consider, in an abstract metric setting, when a *sum* of two nonlinear semigroups can be defined. This can be achieved by means of the operator splitting technique, and a generalization of Trotter-Kato product formulas to nonlinear semigroups is obtained. This lead to the introduction of a sort of semilinear structure on subsets of the family of semigroups defined on a metric space.

References

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HOLONOMIC SYSTEMS ON SYMPLECTIC MANIFOLDS

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WKB microdifferential operators are defined on the cotangent bundle T^*X to a complex manifold X. Open subsets of T^*X are local models for a complex symplectic manifold Y. Even though a globally defined ring of WKB microdifferential operators might not exist on Y, microdifferential modules do make sense. In particular, we will prove the existence of twisted simple holonomic modules on smooth Lagrangian submanifolds of Y.

SUBELLIPTICITY FOR SOME SYSTEMS OF COMPLEX VECTOR FIELDS

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Given $(L_1, ..., L_p), p$ complex vector fields in a neighborhood ω of the origin in $\mathbb{R}^n$, one searches conditions under which one has: $\exists \varepsilon > 0, C > 0$ such that

$$\|u\|_\varepsilon \leq C \left(\sum_{j=1}^{p} \|L_j u\| + \|u\| \right) \quad \forall u \in C_o^\infty(\omega) \tag{1}$$

where $\|\cdot\|$ and $\|\cdot\|_\varepsilon$ are norms in $L^2(\mathbb{R}^n)$ and the Sobolev space H^ε, respectively. In the case when $L_1, ..., L_p$ are real, the celebrated result of L. Hörmander answers this question in terms of the Lie algebra generated by the vector fields.

Also when $p = 1, L_1$ complex, L. Hörmander studied (1), in terms of brackets between $\mathcal{R}eL_1$ and $\mathcal{I}mL_1$, with microlocal results.

Here, we consider the case when $p = n$, and

$$L_j = \frac{\partial}{\partial t_j} + i \frac{\partial \varphi}{\partial t_j} \frac{\partial}{\partial x} \quad in \quad \omega \subset \mathbb{R}^{n+1}_{t,x}, \tag{2}$$

with $\varphi \in C^1(\omega_1; \mathbb{R}), \omega_1$ neighborhood of 0 in $\mathbb{R}^n, \varphi = \varphi(t), \varphi(0) = 0$. To answer (1), $(L_1, ..., L_n)$ given by (2), we have in fact to prove microlocal subellipticity in positive and negative directions dual to the $x-$variable. For that we give a general hypothesis on φ, in order to obtain affirmative answer with a precise $\varepsilon > 0$. H. Maire has studied (1), (2), using combinations of the results of L. Hörmander.

We give classes of functions φ satisfying our general hypothesis and consider in particular, in $\mathbb{R}^3$ (case $n = 2$), homogeneous functions φ. (Nie and Helffer gave some abstract conditions in the case of homogeneous polynomials φ).

COMPRESSIBLE LIQUIDS IN A CAPILLARY TUBE

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We formulate and study a mathematical model for a compressible liquid in a capillary tube. We establish necessary and sufficient conditions for existence and for uniqueness (or for near-uniqueness) of solutions as graphs in tubes of general section, and we provide universal height estimates, depending only on the sectional geometry. We show that solutions exhibit discontinuous dependence properties in containers with corners, analogous to those that are known for the classical capillarity equation.

This work is joint with Maria Athanassenas. It addresses configurations without mass constraint, such as occurs for a vertical capillary tube dipped into an infinite liquid reservoir. A notable result is that if the contact angle exceeds $\pi/2$, existence fails for tubes whose sections are sufficiently narrow. Physically, fluid descends to the bottom of the tube, regardless of its length, a result that could be tested experimentally.

In other work joint with Kevin Luli, we consider a prescribed mass of fluid in a capillary tube closed at the bottom, under a supplementary hypothesis of rotational symmetry. These configurations yield existence for all contact angles less than π, and solutions are unique when the angle does not exceed $\pi/2$.

UNIFORM DECAY OF SOLUTIONS FOR WAVE EQUATIONS IN THE EXTERIOR DOMAIN

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We derive the uniform decay estimate of solutions for wave equations in the outside of compact obstacles. Let $u(t,x)$ be a solution of wave equation in the exterior domain G. Then we can obtain the estimate

$$|u(t,x)| < C(1+|t|)^{-(n-1)/2} \quad \textit{for} \quad (t,x) \in \mathbb{R} \times G$$

without assumptions on the boundary of the domain G.

CORNER SINGULARITIES FOR THE STATIONARY COMPRESSIBLE NAVIER-STOKES SYSTEM

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The solution of the 2-dimensional compressible Navier Stokes system in a plane polygonal domain with specified boundary flow has, in general, singularities at the vertices of the polygon. We shall discuss the problem of determining these singularities, the consequences of these singularities for the resulting flow (in some cases, they "cause" interior irregularities in the solution), and some unresolved questions in this area.

THE EVOLUTIONARY COMPRESSIBLE NAVIER-STOKES SYSTEM ON POLYGONAL DOMAINS

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In this talk I will discuss about the evolutionary compressible Navier-Stokes system on polygonal domains. By extracting the corner singularities by the Laplace equation, the lowest order of the corner singularity of the system is the same as that of the Laplace equation. However, as shown in the heat equation, the coefficient of the singularity, called the stress intensity factor, is expressed by the convolution of some two functions in the time variable. This means that the corner singularity is associated with each time. By a formula of pressure we observe certain propagation of the corner singularity. An increased regularity for the smoother part is shown in a suitable Banach space.

ON THE GLOBAL SOLVABILITY OF KIRCHHOFF EQUATION FOR NON ANALYTIC DATA

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We study the global solvability of the problem:

$$u_{tt} - m\Big(\int_\Omega |\nabla u(x,t)|^2 dx\Big)\,\Delta u \;=\; 0\,, \quad \text{in} \quad \Omega \times [0,\infty) \tag{3}$$

$$u(x,0) = u_0(x)\,, \quad u_t(x,0) = u_1(x)\,, \quad x \in \Omega \tag{4}$$

where Ω is the whole space $\mathbb{R}^n$ $(n \geq 1)$, or a bounded C^2 domain and $u(x,t)$ satisfies the Dirichlet boundary condition $u\big|_{\partial\Omega} = 0$. Assuming

$$m(s) \in C^2\,, \quad m(s) \geq \delta > 0 \quad \text{in} \quad [0,\infty)\,,$$

we give sufficient conditions for the existence of global solutions to problem (1), (2) in terms of the spectral resolution of the initial data $(u_0,\, u_1)$. In particular, we do not require any smallness conditions and we assume only Sobolev type regularity.

HÖRMANDER'S LOCAL PHRAGMÉN-LINDELÖF PRINCIPLE FOR SURFACES IN $\mathbb{C}^n$ CHARACTERIZED BY HYPERBOLICITY CONDITIONS

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Joint work with R.W. Braun and B.A. Taylor

To characterize the surjectivity of linear partial differential operators with constant coefficients on spaces of real analytic functions on convex open sets in $\mathbb{R}^n$, Hörmander used the following local Phragmén-Lindelöf condition:

Definition. For $\xi \in \mathbb{R}^n$ and $r_0 > 0$ let V be an analytic variety in the ball $B(\xi, r_0) \subset \mathbb{C}^n$ which contains ξ. V satisfies $\mathrm{PL}_{loc}(\xi)$ if there exist positive numbers A and $r_0 \geq r_1 \geq r_2$ such that each plurisubharmonic function u on $V \cap B(\xi, r_1)$ which satisfies

$$u(z) \leq 1,\ z \in V \cap B(\xi, r_1) \text{ and } u(z) \leq 0,\ z \in V \cap \mathbb{R}^n \cap B(\xi, r_1)$$

also satisfies

$$u(z) \leq A|\Im z|,\ z \in V \cap B(\xi, r_2).$$

Analytic curves V in $\mathbb{C}^n$ through $\xi \in \mathbb{R}^n$ satisfy $\mathrm{PL}_{loc}(\xi)$ if and only if V is locally hyperbolic at ξ. In general, local hyperbolicity is sufficient but not necessary. New necessary conditions for $\mathrm{PL}_{loc}(\xi)$ were derived in [1]. They were shown to be characterizing for surfaces in $\mathbb{C}^3$. In the lecture a new approch to the problem is presented which leads to a charcterization of the analytic surfaces V in $\mathbb{C}^n$ which satisfy $\mathrm{PL}_{loc}(\xi)$. This characterization is given in terms of limit varieties $T_{\gamma,d}V$ for V with respect to real simple curves γ in $\mathbb{C}^n$ and exponents $d \geq 1$ in combination with hyperbolicity conditions in certain conoids.

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A WEAK NOTION OF PSEUDOCONCAVITY FOR THE TANGENTIAL CAUCHY-RIEMANN SYSTEM

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The talk is a report on recent work, in collaboration with C.D.Hill, on regularity and unique continuation properties for solutions of the tangential Cauchy-Riemann system, under weaker pseudoconcavity assumptions for the Levi form.

ON THE HYPOELLIPTICITY WITH A BIG LOSS OF DERIVATIVES

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I will report on a recent result of C. Parenti and myself about necessary and sufficient conditions for the hypoellipticity with loss of $r + k/2$ derivatives ($r > 0$) of some model classical pseudodifferential operators with symplectic characteristics of multiplicity k.

NONSTATIONARY POISEUILLE FLOW AND THE SOLVABILITY OF NAVIER-STOKES SYSTEM IN A DOMAIN WITH CYLINDRICAL OUTLETS TO INFINITY

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The existence and uniqueness of a solution to the nonstationary Navier–Stokes system having a prescribed flux $F(t)$ in an infinite cylinder is proved. We assume that the initial data and the external forces do not depend on x_3 and find the solution $(\mathbf{u}, p)$ having the following form

$$\mathbf{u}(x,t) = (u_1(x',t), u_2(x',t), u_3(x',t)), \qquad p(x,t) = \tilde{p}(x',t) - q(t)x_3 + p_0(t),$$

where $x' = (x_1, x_2)$. Such solution generalize the nonstationary Poiseuille solutions.

Nonstationary Navier–Stokes system is studied in a two and three-dimensional domains with cylindrical outlets to infinity in weighted Sobolev function spaces. For the two-dimensional problem it is proved that under natural compatibility conditions there exists a solution with prescribed fluxes over cross-sections of outlets to infinity and that this solution tends in each outlet to the corresponding nonstationary Poiseuille flow. The decay rate of the solution is conditioned only by the decay rate of an external force and initial data. In particular, if initial data and external force have compact supports, the solution tends to the corresponding nonstationary Poiseuille flow exponentially. The obtained results are true for arbitrary values of norms of the data (in particular, for arbitrary fluxes) and globally in time. For the three-dimensional case the analogous results are proved either for small data (including fluxes) or for small time intervals.

A LINK BETWEEN LOCAL SOLVABILITY AND PARTIAL ANALITICITY

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This talk deals with the link between the local solvability and "grosso modo" the partial analyticity of the right hand side f of the PDE $P(x,D)u = f, u$ being a Schwartz distribution.The operator P with smooth coefficients is assumed to be locally nonsovable at the origin.Usually,necessary conditions for local solvability of P are proved by imposing a-propriate conditions on the symbol p of P and by violating the well known Hormander's a-priori estimate-necessary condition for local solvability of linear PDO. Certainly,the precise necessary conditions should be imposed on the right hand side f, enabling this way the description of the range of the operator P. In this direction we can mention the paper of Grushin/1972/ on the nonsolvable Mizohata operator,the paper of Greiner-Kohn-Stein/1975/giving complete results on the solvability of Lewy operator and the paper of Ninomiya/1988/dealing with necessary and sufficient condition for local solvability of the Mizohata operator. In this talk we consider second or higher order degenerate parabolic nonsolvable operators and impose necessary conditions on f for local solvability.They are of the form: An integral operator G with explicitely written kernel $\Theta(x,it), \Theta$ being the Weierstrass function,and acting on f should give an analythic function $G(f)$ with respect to the space variable,the time variable being fixed.The proof is reduced to the study of some mixed problem to the backward parabolic operator.

INTUITIVE AND COUNTERINTUITIVE ENERGY FLUX

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The theme of this talk is the computation and estimation of the flux and absolute flux of energy across hypersurfaces for evolution equations. The main surprise is that for smooth rapidly decreasing solutions of the wave equation or Shrödinger equation in dimensions greater than one, the absolute flux of energy across a hyperplane can be arbitrarily large compared to the total energy. For the heat equation the absolute flux never exceeds half the energy.

$L_p - L_q$ DECAY ESTIMATES FOR WAVE EQUATIONS WITH BOUNDED TIME-DEPENDENT COEFFICIENTS

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First we recall to the well-known $L_p - L_q$ decay estimates

$$E(u)(t) \mid_{L_q(\mathbb{R}^n)} \leq C(1+t)^{-\frac{\rho}{2}(\frac{1}{p}-\frac{1}{q})-c\frac{1}{2}} E(u)(0) \mid_{L_{p,N_p}(\mathbb{R}^n)}, \tag{5}$$

for the solutions to

$$u_{tt} - \triangle u + mu + du_t = 0, \;\; u(0,x) = \varphi(x), \;\; u_t(0,x) = \psi(x),$$

where $\rho = n-1$ and $c = 0$ if $m = d = 0$, $\rho = n$ and $c = 0$ if $m > 0$ and $d = 0$ and $\rho = n$ and $c = 1$ if $m = 0$ and $d > 0$. Here $1/p + 1/q = 1$, $1 \leq p \leq 2$, $N_p > n(1/p - 1/q)$.
The goal of the lecture is to discuss such $L_p - L_q$ decay estimates for solutions of the Cauchy problem for strictly hyperbolic equations with bounded time-dependent coefficients of the form

$$u_{tt} - \sum_{k,l=1}^{n} a_{kl}(t) u_{x_k x_l} + m(t)u + d(t)u_t = 0, u(0,x) = \varphi(x), u_t(0,x) = \psi(x). \tag{6}$$

In particular, we are interested in the influence of oscillating behaviour in the coefficients on $L_p - L_q$ decay estimates. For this reason we introduce a classification of oscillations depending on the appearance of a mass or a dissipation. We can observe the so-called *log-effect* which is known from the theory for wave equations with non-Lipschitz coefficients in time. Instead of a loss of regularity we have in our situation a *loss of decay* s_0 depending on the oscillating behaviour itself, thus the desired $L_p - L_q$ decay estimates read as follows:

$$E(u)(t) \mid_{L_q(\mathbb{R}^n)} \leq C(1+t)^{s_0 - \frac{\rho}{2}(\frac{1}{p}-\frac{1}{q})} E(u)(0) \mid_{L_{p,N_p}(\mathbb{R}^n)}. \tag{7}$$

Suitable non-standard symbol classes allow on the one hand the construction of *WKB-solutions* for related ode's with parameter ξ and on the other hand the representation of solutions for (2) by *Fourier multipliers*. Finally, a refined Littman's lemma or *stationary phase method* gives Klein-Gordon or wave decay rates. Most of the results are optimal. We can construct counter-examples basing on *Floquet's theory* which do not allow $L_p - L_q$ decay estimates. Some open problems will complete the lecture.

EXPONENTIAL DECAY FOR SOLUTIONS OF GLOBALLY ELLIPTIC EQUATIONS

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We report on recent results obtained in collaboration with T. Gramtchev, M. Cappiello, E.Buzano, concerning the exponential decay of the solutions of elliptic and hypoelliptic partial differential equations, globally defined in $\mathbb{R}^n$. In particular, we consider linear operators with polynomial coefficients which are elliptic at infinity, as in Shubin, Boggiatto-Buzano-Rodino, Cordes, and others. The solutions in $\mathcal{S}'$ of the homogeneous equations are proved to belong to the so-called Gelfand-Shilov classes, with exponents depending on the characteristic polyhedron of the operator. Similar results are given for semilinear perturbations, under suitable assumptions on the nonlinearity. Proofs are based on a-priori estimates, fixed point techniques in Gelfand-Shilov spaces, construction of parametrices and Fourier integral conjugation.

KINETIC MODELS FOR CHEMOTAXIS WITH THRESHOLD

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In a joint work with Fabio Chalub, we introduce three new examples of kinetic models for chemotaxis, where a kinetic equation for the phase-space density is coupled to a parabolic or elliptic equation for the chemo-attractant, in two or three dimensions. We prove that these models have global-in-time existence and rigorously converge, in the drift-diffusion limit to the Keller-Segel model. Furthermore, the cell density is uniformly-in-time bounded. This implies, in particular, that the limit model also has global existence of solutions.

NECESSARY AND SUFFICIENT CONDITIONS FOR THE SOLVABILITY OF THE EQUATION $\operatorname{div} \underline{w} = p$ $(\underline{w} \in W_0^{1,2}(\mathbb{R}^m)^m, m \geq 1)$.

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While studying weak L^2-solutions to Stokes and Navier-Stokes equations in an infinite cylinder or layer with constant cross section I had to determine the class of those $p \in L^2(\mathbb{R}^m)$ such that the equation $\operatorname{div} \underline{w} = p$ has a unique solution $\underline{w} \in W_0^{1,2}(\mathbb{R}^m)^m$. In addition, there should hold an estimate of the $W_0^{1,2}(\mathbb{R}^m)^m$–norm of the solution $\underline{w}$ by certain quantities depending on p.

SCHAUDER ESTIMATES FOR EVOLUTION GENERALIZED STOKES PROBLEM

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We consider the initial-boundary value problem

$$\mathbf{v}_t + \mathcal{A}(x,t,\frac{\partial}{\partial x})\mathbf{v} + \nabla p = \mathbf{f}(x,t), \qquad \nabla\cdot\mathbf{v} = 0, \quad (x,t)\in Q_T \equiv \Omega\times(0,T), \tag{1}$$

$$\mathbf{v}(x,0) = \mathbf{v}_0(x), \quad \mathbf{v}(x,t)\Big|_{x\in S} = \mathbf{b}(x',t), \tag{2}$$

in n-dimensional bounded domain Ω whose boundary $S\in C^{2+\alpha}$, $\alpha\in(0,1)$, can consist of several connected components. Here unknown are the vector field $\mathbf{v}(x,t) = (v_1,,,v_n)$ and the function $p(x,t)$. By $\mathcal{A}$ we mean a second order strongly elliptic operator with real coefficients whose principal part $\mathcal{A}_0$ satisfies the condition

$$C^{-1}|\xi|^2|\eta|^2 \le \mathcal{A}_0(x,t,i\xi)\eta\cdot\eta \le C|\xi|^2|\eta|^2, \quad \forall\xi,\eta\in R^n\setminus\{0\}$$

with the constant $C>0$ independent of x,t. Equation of the type (1) arises in the linearization of equations of motion of a certain class of non-Newtonian liquids. The Stokes system corresponds to the case $\mathcal{A} = -\nu I\Delta$. We prove that arbitrary solution of problem (1),(2) $\mathbf{v}\in C^{2+\alpha,1+\alpha/2}(Q_T)$, $\nabla p\in C^{\alpha,\alpha/2}(Q_T)$ satisfies the two-sided inequality

$$c(T)^{-1}\Big(|\mathbf{v}|_{C^{2+\alpha,1+\alpha/2}(Q_T)} + |\nabla p|_{C^{\alpha,\alpha/2}(Q_T)}\Big) \le |\mathbf{f}|_{C^{\alpha,\alpha/2}(Q_T)} + |\mathbf{v}_0|_{C^{2+\alpha}(\Omega)}$$

$$+|\mathbf{a}|_{C^{2+\alpha,1+\alpha/2}(\Sigma_T)} + \sum_{k=1}^{n}[\mathcal{R}_k(\mathbf{a}_t\cdot\mathbf{n})]_{t,\Sigma_T}^{(\alpha/2)}$$

$$\le c(T)\Big(|\mathbf{v}|_{C^{2+\alpha,1+\alpha/2}(Q_T)} + |\nabla p|_{C^{\alpha,\alpha/2}(Q_T)}\Big)$$

where $C^l(\Omega)$ and $C^{l,l/2}(Q_T)$ are isotropic and anisotropic Hölder spaces, and $\mathcal{R}_k$ are the Riesz operators on the surface S. Under necessary compatibility conditions for the data, the related existence theorem for problem (1), (2) is proved.

A REGULARITY RESULT FOR NONLINEAR WEAKLY HYPERBOLIC SYSTEMS

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We discuss the propagation of analytic regularity for Gevrey solutions to semilinear systems with space dependent coefficients. The technique employed is based on the theory of quasi-symmetrizers. This is a joint research with G. Taglialatela.

WHY ANALITICITY FOR PDE's? AND WHERE?

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This talk will consider partial differential equations arising (originally) in connection with functions of several complex variables and discuss the regularity (analyticity) of their solutions and some consequences.

SYMMETRIZATION OF COMPLEX HYPERBOLIC SYSTEMS AND REDUCED DIMENSION

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In the first part, we consider a complex strongly hyperbolic linear system of first order with constant coefficients. We state that, if the reduced dimension of the system is enough large, the system is hermitian in convenient basises. The second part is a common work with T. Nishitani. The system has variable coefficients and is linear or quasi linear. We assume, for frozen coefficients, the precedent assumptions and we state that the system is regularly "symmetrizable"; that means the convenient matrix of change of basis is smooth in x (and in the unknown function, in the quasi linear case).

SPLITTING OF DIFFERENTIAL COMPLEXES

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We discuss the splitting of a short exact sequence

$$0 \longrightarrow G \longrightarrow E \xrightarrow{S} F \longrightarrow 0$$

of nuclear Fréchet spaces, where F need not admit a continuous norm, i.e. E or F may be spaces of smooth functions or sections of smooth bundles on an open subset of $\mathbb{R}^n$ or on a σ-compact C^∞-manifold. If E is such a space then we give a necessary and sufficient condition for the splitting of the sequence, in terms of a condition on S and of linear topological invariants (Ω_{loc}) and (DN_{loc}). This is used to study the splitting of differential complexes and to give a condition for splitting in terms of $P(D)$-convexity with bounds, so generalizing results of R. Meise, B. A. Taylor, D. Vogt and V. P. Palamodov (see P. Domański, D. Vogt, *A splitting theorem for the space of smooth functions*, J. Funct. Anal. **153** (1998), 203-248 and D. Vogt, *Splitting of exact sequences of Fréchet spaces in the absence of continuous norms*, J. Math. Anal. Appl. 297 (2004), no. 2, 812–832).

ON SOME LIOUVILLE TYPE PROBLEMS

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We study the problem of existence of nontrivial bounded solutions to:

$$-\partial_{x_i}(a_{ij}(x)\partial_{x_j}u) + a(x)u = 0 \qquad \text{in } \mathcal{D}'(\mathbb{R}^k),$$

when a_{ij} and a are bounded functions in $\mathbb{R}^k$.

AN INNOVATIVE USE OF KOBAYASHI METRIC IN INTEGRAL GEOMETRY

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With L. Baracco and A. Tumanov

Let $D_2 \subset D_1$ be relatively compact strongly convex domains in C^n and let f be a continuous function on ∂D_1 which extends holomorphically along analytic discs with boundary in ∂D_1 and which are tangent to ∂D_2. Is then f actually holomorphic in D_1? This problem is standing since long time. A positive answer, maybe the unique so far, is when D_1 and D_2 are balls with the same centers (cf. " Function Theory in the unit ball" by W. Rudin). The proof is founded in harmonic analysis and seems unsuitable for generalization. We take, instead, the view point of complex analysis and CR geometry. First, if one wants to deal with general convex domains, one has to clarify which are the suitable "discs" which replace the straight complex lines in the case of the ball. Now, it appears that the right geometric approach is the one of the "stationary discs" that is the geodesics in the Kobayashi metric. These discs carry a significant geometric meaning, since they can be viewed as discs in the cotangent bundle T^*C^n attached to the conormal bundle $T^*_{\partial D_1}C^n$. The boundary being Levi-nondegenerate, this bundle has no complex tangencies (it is "totally real"). By "lifting" the function f to these discs one obtains in a natural way a CR function overe a wedge in T^*C^n with such a real edge to which one can apply the standard theory of holomorphic extension of CR functions. But this construction turns out to be illusory. There was no lift at all, and what we holomorphically extended was the function f itself.