The Cauchy Problem for a Class of p-evolution Equations.

Alessia Ascanelli

We study p-evolutive equations of order $m \geq 2$ with coefficients depending both on t and x, not regular with respect to the time variable.
We find the sharp regularity in time for the coefficients in order to have well posedness in H^∞ of the Cauchy Problem.
We consider the case of continuous in time coefficients having the first time-derivative that breaks down at a point t_0, say $t_0 = 0$.
The case $m = 2$ has already been studied by Cicognani and Colombini; here we generalize the result to operators of higher order.

On the $W^{2,s}$ Regularity to some Boundary Value Problems for Flows with Gradient Dependent Viscosity.

Hugo Beirão da Veiga

We present some regularity results in L^s spaces, up to the boundary, for the second derivatives of the velocity and for the gradient of the pressure, to solutions of flows with Ladyzhenskaya's shear dependent viscosity and slip or non slip boundary conditions.

On the Solvability of the Free Boundary Problems Describing Phase Transition with Supercooling Effect

Galina I. Bizhanova

Let $\Omega \in R^n, n \geq 2$, be bounded domain with boundary S, let $\gamma(t),\ t \in [0,T]$, be closed surface, divided Ω onto two subdomains $\Omega_1(t)$ and $\Omega_2(t)$ such, that $\partial\Omega_1(t) = \gamma(t) \bigcup S,\ \partial\Omega_2(t) = \gamma(t)$. At the initial moment we have $\gamma(0) = \Gamma,\ \Omega_m(0) = \Omega_m,\ m = 1,2$.

Let $L_m(x,t,\partial_x,\partial_t)$ be second order parabolic operator

$$L_m = \sum_{i,j=1}^{n} a_{ij}^{(m)}(x,t)\partial_{x_i x_j}^2 + \sum_{i=1}^{n} a_i^{(m)}(x,t)\partial_{x_i} + a^{(m)}(x,t).$$

We consider two free boundary problems with unknown functions $u_1(x,t), u_2(x,t)$ and $\gamma(t)$, satisfying parabolic equations

$$L_m u_m = 0,\ x \in \Omega_m(t),\ t \in (0,T],\ m = 1,2,$$

initial and boundary conditions

$$u_m|_{t=0} = u_{0m}(x),\ x \in \Omega_m,\ m = 1,2,\ \gamma|_{t=0} = \Gamma,\ u_1|_S = p(x,t)$$

and conditions on free boundary $\gamma(t)$
in the Problem I

$$u_1 = u_2,\ u_1 = -\beta V_\nu,\ \lambda_1(x,t)\partial_\nu u_1 - \lambda_2(x,t)\partial_\nu u_2 = \kappa V_\nu, t \in (0,T]$$

and in the Problem II

$$u_1 = u_2,\ u_1 = -\beta V_\nu,\ \lambda_1(x,t)\partial_\nu u_1 - \lambda_2(x,t)\partial_\nu u_2 = 0,\ t \in (0,T],$$

where ν– outer normal to $\Omega_2(t),\ V_\nu$-the velocity of $\gamma(t)$ in the direction of the normal $\nu,\ \beta, \kappa$–positive constants.

The problems I and II describe the physical process of melting or solidification of the substance, when liquid phase may have temperature lower then the temperature of phase transition staying in liquid condition.

The unique solvability of the problems I and II locally in time in the Holder and weighted Holder spaces is proved, the estimates for the solution are obtained.

A New Approach to Deriving Global Estimates for Strong Solutions of the Navier-Stokes Equations

Mikhail E. Bogovskii

A new approach, based on solving a certain auxiliary problem, makes it possible to derive a global estimate of the Dirichlet integral implying its being bounded in time for the nonlinear initial boundary value problem

$$\begin{cases} v_t + (v, \nabla)v - \nu\Delta v + \nabla p = f(x,t), \\ \operatorname{div} v = 0,\ (x,t) \in Q_{_T} = \Omega \times (0,T), \\ v\big|_{t=0} = v^0(x),\ \operatorname{div} v^0 = 0,\ x \in \Omega, \\ v\big|_{\partial\Omega} = 0,\ t \in (0,T),\ v^0\big|_{\partial\Omega} = 0, \end{cases} \tag{1}$$

in a bounded domain $\Omega \subset \mathbb{R}^3$ with boundary $\partial\Omega$ of the class C^2. A solution of problem (1) is being treated here as an ordered pair of the class

$$\{v, \nabla p\} \in W_{2,x,t}^{2,1}(Q_{_T}; \mathbb{R}^3) \times L^2(Q_{_T}; \mathbb{R}^3) \tag{2}$$

that involves the anisotropic Sobolev space $W_{2,x,t}^{2,1}$ of $\mathbb{R}^3$-valued vector functions $v(x,t)$ having square-summable weak derivatives v_t, $D_x^\alpha v \in L^2(Q_{_T}; \mathbb{R}^3)$ for all multiindices α such that $|\alpha| \leq 2$. A solution of the class (2), usually referred to as strong, requires the right-hand side $f \in L^2(Q_{_T}; \mathbb{R}^3)$ and the initial data v^0 from the Sobolev space $W_2^1(\Omega; \mathbb{R}^3)$ of $\mathbb{R}^3$-valued vector functions $w(x)$ having square-summable weak derivatives $D_x^\alpha w \in L^2(\Omega; \mathbb{R}^3)$ for all α such that $|\alpha| \leq 1$. No restrictions are imposed either on the positive numbers ν and T or on the values of appropriate norms of f and v^0.

To derive the global estimate of the Dirichlet integral, an auxiliary initial boundary value problem with two parameters $s \in [0,1]$ and $\lambda \geqslant 0$ is introduced

$$\begin{cases} u_t + \eta_{_T}(U^\lambda, \nabla)u - \nu\Delta u + \nabla q = sf(x,t), \\ \operatorname{div} u = 0,\ (x,t) \in Q_\infty = \Omega \times (0,\infty), \\ u\big|_{t=0} = sv^0(x),\ u\big|_{\partial\Omega} = 0, \end{cases} \tag{3}$$

with v^0 the same as in (1), while f borrowed from (1) is extended by zero, i.e. $f(x,t) = 0$ for $t > T$. Notation $\eta_{_T}(t)$ stands for some smooth

cutoff function such that $\eta_T(t) = 1$ when $t \leqslant T$. A vector function U^λ is defined as

$$U^\lambda(x,t) = u\big(x, t + \varphi(u) - \lambda\big),\ (x,t) \in Q_\infty\,,$$

with $u(x,t) = sv^0(x)$ for $t < 0$, where φ denotes the nonlinear functional

$$\varphi(u) = \max_{t \geqslant 0} \int_\Omega |\nabla u(x,t)|^2 dx.$$

Given any appropriate f and v^0 in (1), auxiliary problem (3) has a solution $\{u, \nabla q\}$ of the class (2) for all $s \in [0,1]$ and $\lambda \geqslant 0$. Whether this solution is unique stays unclear. But there is only one maximal solution, i.e. solution with the maximum value of the functional $\varphi(u)$ corresponding to given f, v^0, s, λ. The new approach basically rests upon the fact that the maximal solution is continuous in parameters $s \in [0,1]$ and $\lambda \geqslant 0$. Namely, the continuity in $s \in [0,1]$ traps the values of $\varphi(u)$ within a segment the length of which depends on f, v^0, λ, whereas the continuity in λ guarantees the existence of the value $\lambda = \varphi(u)$ which turns $\{u, \nabla q\}$ into a solution $\{v, \nabla p\}$ of problem (1).

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Optimal Well-posedness of the Cauchy Problem for Evolution Equations with C^N Coefficients

Massimo Cicognani, Ferruccio Colombini

We deal with the Cauchy Problem for a $2-$evolution operator of Schrödinger type with C^N coefficients in the time variable, $N > 2$. We find the Levi conditions for well-posedness in Gevrey classes of index $1/2 + N/4$ which is the best possible as we show by means of counterexamples.

Models and Problems for Phase Transitions with Microscopic Movements

Pierluigi Colli

A class of phase transition models based on the Fremond approach of microscopic movements and consistent with thermodynamics will be reviewed. The corresponding problems couple two nonlinear differential equations, in terms of absolute temperature and phase parameter, that look rather complicate and difficult to handle in their full generality. Some existence results for reduced or simplified versions of the field equations will be outlined.

The Riemann Problem for Metastable Reversible Reactive Flows

Andrea Corli, Haitao Fan

A hyperbolic model for dynamic phase transitions is studied. The model involves three phases: liquid, vapor, and a mixture of them. Metastable regions are present both in the liquid and in the vapor phase.

Results on the behavior of traveling wave profiles of the model, involving viscosity, species diffusion and relaxation, are obtained. These behaviors are consistent to physical intuitions. Admissibility criteria (kinetic relations) that mimic the behavior of traveling wave profiles are then proposed. Admissible basic waves of the model are liquefaction, evaporation and isobaric waves, in addition to Lax shock and rarefaction waves. Based on these waves, solutions of the Riemann problem for the model are constructed for general Riemann initial data. Most of the physical phenomena are embodied in the solver. For some Riemann initial data solutions are expected to be nonunique: which solutions actually appear depends on whether nucleation already occured or not. The model admits both solutions, as it should.

The model also has two other type of waves, collapsing and explosion waves. More complicated solutions involving these two waves are also proposed and discussed.

Stability of an infinite flexible beam under a viscous fluid flow with an exponential profile

Irina V. Denisova

We consider a stationary flow problem in the half-plane. We study the stability of an infinite viscous-elastic beam which is flowed by a viscous incompressible liquid. The Navier - Stokes system is linearized about a flow velocity vector with the exponential profile of velocity. The adhesion boundary condition is imposed at the beam whose deflection from the zero state is subjected to the elasticity equation. After the Fourier - Laplace transform, an explicit solution to the problem is found in the form of convergent rows. The roots of the dispersion equation which characterize the stability of the system are calculated by computer. The real part of these roots depends on the flow speed, the viscosity and the beam characteristics. These dependences are studied when the dynamical loss of stability arises.

Free Boundary Problem for a Layer of Inhomogeneous Fluid

Elena Frolova, Mariarosaria Padula

We consider a horizontal layer of viscous fluid with variable density and free upper surface, in the presence of gravitational force and surface tension. At first, we prove that the rest state with zero velocity can have only flat upper surface, then we exhibit rest states with different pressures and densities in a fixed cell of layer, having given volume V, and filled by inhomogeneous fluid with the given mass M. As a consequence of nonuniqueness of rest state, there is no hope to obtain asymptotic stability of any basic rest state.

Recently, M.Padula and V.A.Solonnikov [1] proved the exponential decay of the perturbations to the rest state for the case when the layer is filled by incompressible homogeneous fluid or by isothermal fluid. In our opinion, the model of inhomogeneous fluids has physical drows back, because this model doesn't allow any decay to zero for perturbations of a basic density and does not admit asymptotic stability of the rest state. We set the free boundary problem for a inhomogeneous incompressible fluid in a way similar as that proposed in [1] and assume the existence of global smooth solution. In the class of linear basic densities we provide a control of L^2-norms of perturbations for all the variables of the motion (stability in the mean) and prove that the velocity decays to zero along a sequence of times. Furthermore, for the sufficiently small perturbations to initial density, we show that more regular norms of perturbations to velocity, density, and height are bounded at any moment of time. The uniqueness and stability method employs the ideas developed in [1], where a generalized energy functional is introduced. However, in this case, the equation of conservation of mass can no more be combined with dissipative effects present into the momentum equation, because the pressure is an independent term, and the dissipative term corresponding to variation of density does not appear in the energy inequality.

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Sharp Estimates for Green's Functions: Singular Cases

Maria Giovanna Garroni

The purpose of this lecture is to present a survey of some results contained in a number of papers and two books, over the years 1984-2002, concerning the construction and the properties of the Green function for parabolic second-order operators, under different "non-regular" hypotheses. In this presentation, I will consider only the most complex case: i.e., the problems with oblique boundary conditions.The first result, obtained in [1], deals with the case when the coefficients of the boundary operator are only Holder continuous. The main difficulties here arise from the impossibility, under these weak hypotheses, of obtaining estimates with global Holder norms for the highest-order derivatives. The second type of results is concerned with a large class of parabolic integro-differential operators that arise from diffusion processes with jumps, with either regular or weak hypotheses on the boundary operator. To our knowledge, the Green function is generally sought in the form of the sum of two terms: a principal term (with the highest singularity) and an additional less singular term. It is well known that for differential operators the exponential term in a kernel of the heat type plays an essential role in construction this additional term and in establishingsharp estimates. Since the integro-differential operator propagates the classical singularity at the origin, estimates of the heat-kernel type cannot exist. Hence, we have been forced to identify the key properties needed for the construction of the additional term. By means of these properties, which can be expressed through some seminorms, we define a decreasing family of Banach spaces, referred to as Green spaces. For the main results and applications we refer to [2,3]. Finally, the case of the heat equation with oblique constant conditions in a dihedral angle is studied. In this case, convenient weighted Sobolev spaces are introduced, where the weight is the distance from the vertex. These spaces allow to construct the Green function and to obtain the estimates that describe its behaviour at the points where it has singularities. Much more complicate is to obtain estimates that guarantee the exponential decay of the Green function and of its derivatives as in the classical parabolic problems in regular domains. These results are contained in [4-7].

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On some phase transition models of Penrose-Fife type.

Gianni Gilardi

We deal with a system of two nonlinear evolution PDEs modelling phase transitions and ensuring thermodynamical consistency. The assumptions on the heat flux law appearing in the energy balance equation cover a wide range of possible behaviours at low and high absolute temperatures. The phase dynamics is ruled by a 2nd or 4th order equation according to the so-call nonconserved and conserved cases. Various boundary conditions are considered and well-posedness results are presented.

On Compressible Bénard Problem

Giovanna Guidoboni

Bénard problem consists in the investigation of stability of rest state of a layer of fluid heated from below.
Introducing a generalized energy functional which contains only the L^2 norm of the perturbations we study the non-linear stability of rest state for a layer of compressible fluid. Euler-Lagrange equations are written for the maximum problem and the numerical computation of the eigenvalues is in progress.
The stability of rest state for a layer heated from above is trivial in the incompressible case (*Buossinesq approximation*) while it is an open problem in the compressible case both in the linear and non-linear analysis. We present a partial result in this direction. Neglecting the thermal conductivity, we can prove that for a layer of compressible fluid

- rest state is always stable, when the layer is heated from above;
- rest state is stable if the imposed temperature gradient β is less then g/c_p, knwown as adiabatic gradient, when the layer is heated from below.

A Sharp Attainment Result for Nonconvex Variational Problems

Marcello Guidorzi, Pietro Celada, Giovanni Cupini

We consider the problem of minimizing autonomous, multiple integrals like

$$\min \left\{ \int_\Omega f(u, \nabla u)\, dx : u \in u_0 + W_0^{1,p}(\Omega) \right\} \qquad (\mathcal{P})$$

where $f\colon \mathbf{R}\times\mathbf{R}^N \to [0, \infty)$ is a continuous, possibly nonconvex function of the gradient variable ∇u. Assuming that the bipolar function f^{**} of f is affine on each connected component of the detachment set $\mathcal{D} = \{(\eta, \xi) : f^{**}(\eta, \xi) < f(\eta, \xi)\}$, we prove attainment for $(\mathcal{P})$ under mild assumptions on f and f^{**}. We present examples that show that the hypotheses on f and f^{**} considered here for attainment are essentially sharp.

Elliptic Problems with Nonlocal Conditions near the Boundary

Pavel I. Gurevich

In the talk, we will consider nonlocal elliptic boundary value problems in bounded domains. We will discuss the most difficult case where the support of nonlocal terms intersects with the boundary by some set K. In such a situation, solutions to nonlocal problems may have power singularities near the points of the set K.

In [1], these problems were considered in the weighted Kondrat'ev spaces. We investigate nonlocal problems in Sobolev spaces with no weight. On a model example we will show that the corresponding operator in Sobolev spaces may be neither invertible nor even Fredholm. It turns out that the solvability of nonlocal problems is related to (1) location of eigenvalues of some auxiliary 1-dimensional nonlocal problem with parameter, (2) structure of eigenvectors and associated vectors corresponding to some eigenvalues, (3) explicit algebraic conditions on the differential operator and nonlocal boundary value operators.
We will give explicit necessary and sufficient conditions for the nonlocal problem to be Fredholm.
This research has been supported by Russian Foundation for Basic Research (grant 02-01-00312) and by INTAS (grant YSF 2002-008).

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Mathematical Problems of Rotating Turbulence

Alex S. Mahalov, Basil P. Nicolaenko

In this talk we review recent mathematical results on three-dimensional rotating turbulence. On the mathematical side, rotating turbulence is described by 3D Navier-Stokes Equations with initial data characterized by uniformly large vorticity. We prove existence on infinite time intervals of regular solutions to the 3D Navier-Stokes Equations for such class of large initial data both in infinite R^3 and bounded cylindrical domains. There are no conditional assumptions on the properties of solutions at later times, nor are the global solutions close to some 2D manifold. The approach is based on fast singular oscillating limits, nonlinear averaging and cancellation of oscillations in the nonlinear interactions for the vorticity field. With nonlinear averaging methods in the context of almost periodic functions, resonance conditions and a nonstandard small divisor problem, we obtain fully 3D limit resonant Navier-Stokes equations. We establish the global regularity of the latter without any restriction on the size of 3D initial data and with the help of strong convergence theorems bootstrap this into the global regularity of the weak solutions of 3D Navier-Stokes Equations with weakly aligned uniformly large vorticity at $t = 0$. Other mathematical questions arising in the theory of rotating turbulence will be discussed and experimental observations on 3D rotating turbulence by Hopfinger et al. will be used to guide physical and mathematical intuition.

Asymptotic Proprieties of Solutions of the Navier-Stokes Equations in Half-Space

Paolo Maremonti

For the Navier-Stokes system we consider the following initial boundary value problem in half-space:

$$\begin{aligned} &u_t - \Delta u + (u\cdot\nabla)u = -\nabla\pi, \quad \nabla\cdot u = 0, \text{ on } \mathbb{R}^n_+ \times (0,T),\\ &u(x',t) = 0, \text{ on } \mathbb{R}^{n-1}\times(0,T), \quad \lim_{|x|\to\infty} u(x,t) = 0,\\ &u(x,0) = a(x), \text{ on } \mathbb{R}^n_+, \end{aligned} \tag{4}$$

where $u_t = \frac{\partial}{\partial t}u$, $(u\cdot\nabla)u = u_k\frac{\partial}{\partial x_k}u$, $n \geq 3$, $x' = (x_1,\cdots,x_{n-1},0)$. We introduce the sets of functions:

$$\mathcal{M}(\mathbb{R}^n_+) = \left\{h(x) \in C(\overline{\mathbb{R}}^n_+), h(x') = 0, \nabla\cdot h(x) = 0, |h(x)| = o(1)\right\},$$

and, for any $s \in (0, n-1)$,

$$\mathcal{M}^s_{|x|}(\mathbb{R}^n_+) = \left\{h(x) \in \mathcal{M}(\mathbb{R}^n_+), \text{with } |h(x)| \leq \frac{H_0}{(1+|x|)^s} \text{ for some } U_0 > 0\right\}.$$

We are interested to prove

Theorem - *For any $a(x) \in \mathcal{M}^s_{|x|}(\mathbb{R}^n_+)$, there exists a unique solution (u,π) of system (4) such that, for some $T > 0$ and for any $\eta \in (0,T)$, for $\alpha \in (0,1)$,*

$$\begin{aligned} &u \in C(0,T;C(\overline{\mathbb{R}}^n_+)) \cap C(\eta,T;C^{2,\alpha}(\overline{\mathbb{R}}^n_+)),\\ &u_t, \nabla\pi \in C(\eta,T;C^{0,\alpha}(\overline{\mathbb{R}}^n_+)),\\ &\lim_{t\to 0^+} u(x,t) = a(x), \forall x \in \mathbb{R}^n_+. \end{aligned}$$

Moreover, there exist two suitable constants b and k_0, such that:

if $s \in (0,1)$, we have $(4bk_0A_0)^{-1} \leq T^{\frac{1}{2}-\frac{s}{2}}$ and for $k \in [0,s]$

$$|u(x,t)| \leq \frac{k_0A_0}{1-2bk_0A_0t^{\frac{1}{2}-\frac{s}{2}}}\frac{1}{(1+|x|)^{s-k}}\frac{1}{(1+t)^{\frac{k}{2}}}, \quad \forall (x,t) \in \mathbb{R}^n_+\times[0,T); \tag{5}$$

if $s \in [1,n-1)$ e $A_0 < (4bk_0)^{-1}$, then $T = \infty$ and for $k \in [0,s]$

$$|u(x,t)| \leq \frac{k_0A_0}{1-2bk_0A_0}\frac{1}{(1+|x|)^{s-k}}\frac{1}{(1+t)^{\frac{k}{2}}}, \quad \forall (x,t) \in \mathbb{R}^n_+\times[0,\infty). \tag{6}$$

Constants b and k_0 are independent of $a(x)$.

Regularity of Vectorial Integrals with non Standard Growth

Elvira Mascolo

In this talk we present some results on the regularity of vectorial functionals of Calculus of Variations. In particular, we consider integral functionals with density energies satisfying (p-q) and general growth conditions and we obtain that the minimizers are Lipschitz continuous.

Stabilizing Effects in Dynamical Systems: Linear and Nonlinear Stability Conditions

Gaetano Mulone

Many dynamical systems, in particular fluid-mechanics and convection problems, exhibit stabilizing effects which are confirmed by the experiments and by the linearized instability method. The aims of this talk are:
i) To study nonlinear stability of the zero solution of a dynamical system

$$\begin{cases} \mathcal{U}' = L\mathcal{U} + N\mathcal{U} \\ \mathcal{U}(0) = \mathcal{U}_0 \end{cases}$$

with a linear operator which splits in a symmetric and a skewsymmetric part and to give (for particular systems) an operative method (which rests upon the eigenvalues - eigenvectors method in ODE) to build a good Lyapunov function in order to obtain necessary and sufficient stability conditions. The method is applied to a reaction-diffusion system and to thermodiffusive convection of a mixture heated and salted from below.
ii) To show - in the linear case - the stabilizing effect of the magnetic field on the onset of convection for a compressible Navier-Stokes-Fourier fluid bounded by two horizontal planes and heated from below.

3D Euler Equations with Weakly Aligned Large Initial Vorticity in Bounded Domains

Basil P. Nicolaenko, Alex S. Mahalov

We consider the 3D incompressible Euler Equations with initial data characterized by uniformly large vorticity in bounded cylindrical domains. We prove existence on arbitrary large time intervals of regular solutions with large kinetic energy to the 3D Euler Equations for a class of initial data which induce large vortex stretching. There are no conditional assumptions on the properties of solutions at later times, nor are the global, non-axisymmetric solutions close to any 2D manifold; the ratio of the large enstrophy to the large kinetic energy is of order one. The approach is based on fast singular oscillating limits, nonlinear averaging and cancellation of oscillations in the nonlinear interactions for the vorticity field. With nonlinear averaging methods in the context of almost periodic functions and resonance conditions, we obtain fully 3D limit resonant Euler equations. For almost all cylindrical domains, we establish the long time regularity of solutions to the resonant equations without any restriction on the size of 3D initial data ; we bootstrap this into the long time regularity of Euler Equations with uniformly large vorticity weakly aligned along the cylinder axis at t=0. The geometry of the curl operator with boundary conditions, together with vector potential inverse operators, are essential to obtain cancellation of vorticity oscillations in a bounded domain. The specific difficulties of the 3D Euler case versus the Navier-Stokes case are discussed , with emphasis on the pitfalls of harmonic components of fast oscillating pressure fields.

On Compactness Properties of Solutions to the Compressible Navier-Stokes Equations

Pavel I. Plotnikov, Jan Sokolowski

We investigate the compactness properties of solutions to the time - discretized Navier-Stokes equations of compressible flows in three space dimension on condition that the adiabatic constant satisfies $\gamma \geq 1$. It is supposed that the velocity of fluid $\mathbf{u}$ and the density ρ satisfy the equations

$$\alpha\rho\mathbf{u} + \operatorname{div}(\rho\mathbf{u}\otimes\mathbf{u}) + \nabla(\rho)^\gamma = \rho f + \operatorname{div} G, \tag{7a}$$

$$\alpha\rho + \operatorname{div}(\rho\mathbf{u}) = h, \tag{7b}$$

where G is a viscous stress tensor, α is a positive constant, f and $h \geq 0$ are continuous given functions.

The main result is the following theorem on compactness properties of solutions to equation (7). Let us consider the sequence of solutions $(\mathbf{u}_n, \rho_n)$ to (7) defined in a bounded domain $\Omega \in \mathbb{R}^3$ and satisfying the energy inequality

$$\|\mathbf{u}_n\|_{H^{1,2}(\Omega)} + \|\rho_n\|_{L^\gamma(\Omega)} + \int\limits_\Omega \rho_n|\mathbf{u}_n|^2 dx \leq c < \infty \quad \text{for all} \quad n \geq 1.$$

Suppose that $\rho_n \to \rho$ weakly in $L^\gamma(\Omega)$, $\mathbf{u}_n \to \mathbf{u}$ weakly in $H^{1,2}(\Omega)$ and $\rho_n\mathbf{u}_n\otimes\mathbf{u}_n \to \mathcal{M}$ star weakly in the space of Radon measures, where $\mathcal{M}$ denotes 3×3 matrix- valued Radon measure in Ω.

Theorem 1 *If the adiabatic constant satisfies the inequality $\gamma > 1$, then $\mathcal{M} = \rho\mathbf{u}\otimes\mathbf{u}$ and $\rho_n^\gamma \to \rho^\gamma$ in $\mathcal{D}'(\Omega)$. In the isothermal case $\gamma = 1$ the measure $\mathcal{M}$ has the representation*

$$\mathcal{M} = \rho\mathbf{u}\otimes\mathbf{u} + \mathcal{S}, \tag{8}$$

in which the defect measure is defined by the equality

$$\int\limits_\Omega \varphi(x) : d\mathcal{S}(x) = \int\limits_{\Omega_{sing}} \mathbf{s}(x)\otimes\mathbf{s}(x) : \varphi(x)\sigma(x) d\mathcal{H}^1. \tag{9}$$

Here Ω_{sing} is a Borel set, which every compact subset is $(\mathcal{H}^1, 1)$ rectifiable , $\mathbf{s}(x)$ is the unit tangent vector to Ω_{sing} at point x, $\mathcal{H}^1$ is the one-dimensional Hausdorff measure and σ is a non-negative, bounded function.

Modelling Oxygen Perfusion of a Living Tissue by a Network of Capillaries

Mario Primicerio, Andro Mikelic

The aim of this paper is to present and discuss a mathematical model describing the exchange of oxygen between a network of capillary tubes and a living tissue surrounding them. It is assumed that the capillaries are parallel and that blood flows within them at a prescribed velocity $u(t)$. Oxygen is transported by blood in two forms, either dissolved in blood plasma or bound to erythrocytes (red cells) in form of haemoglobin. In the living tissue, partial pressure of oxygen is assumed to obey a non-linear diffusion equation (assuming an instantaneous equilibrium between directly available and stored oxygen, as foreseen e.g. by a law of type Michaelis-Menten) with a non-linear consumption term due to metabolism. It is reasonable to neglect radial variation of the relevant quantities within the capillaries, and to assume that oxygen flux on the walls is proportional to the jump of partial pressures, i.e. of concentrations of the fraction of the gas that is free to diffuse. This turns out to model the exchange as a concentrated capacity for the oxygen in the capillaries and as a boundary condition for the oxygen diffusing in the tissue. The discussion of the well posedness of the mathematical problem is based on the use of compactness arguments and suitable a-priori estimates. The results are global in time. A further step is homogenization of the problem both in the periodic and in the random case.

Estimates of Deviations from Exact Solutions for Boundary-Value Problems with Incompressibility Condition

Sergey I. Repin

The talk is focused on deriving estimates of the difference between exact and approximate solutions of boundary–value problems arising in the theory of viscous incompressible fluids. In the classical Stokes problem, such estimates include three terms that account the range of incompatibility in the basic equations and in the divergence–free condition. The latter, presents the most complicated task. It is shown that the desired estimate for this term can be derived on the basis of the results obtained by O.A. Ladyzhenskaya and V.A. Solonnikov when analyzing solvability of the Stokes problem.

Analytic and Gevrey Solutions of Non-Linear Partial Differential Equations.

Luigi Rodino

Several partial differential equations from Mathematical Physics present nonlinearities and higher order multiplicity of the characteristics. Natural functional frame for them is the class of the analytic-Gevrey functions, allowing well-posedness of the Cauchy problem or local solvability. In this connection, we review some known results and present some new contributions.

Dynamics of Droplets in an Agitated Dispersion with Multiple Breakage

Fabio Rosso, Antonio Fasano

Some years ago we proposed a model (see [2,3]) for the dynamics of droplets including a constraint on the droplet size in a consistent way and pointing out that this requires the presence of a third physical mechanism in the evolution of the system, called *volume scattering*. This effect consists in the immediate decay by rupture of a parent droplet resulting from coalescence and exceeding the threshold value v_{m} so that all daughters remain in the allowed size range. Scattering is represented by a specific operator (with gain and loss terms) in the balance equation which adds to classical coalescence and breakage operators. The main advantage of this approach is that it is based on natural assumptions, reflecting the real physics. Also the mathematics appears to be simpler since subtle questions regarding summability in unbounded domains are automatically eliminated. The resulting model consists in an initial value problem for a Boltzmann–like equation for the function $f(v,t)$. Our first contribution dealt with the somehow artificial case of *binary* rupture events: this means that any parent droplet involved within breakage or scattering produces exactly *two* daughters. In [2,3] we proved that the problem is well posed under rather general hypotheses and with a *bounded* fragmentation kernel. The extension to the unbounded case was subsequently worked out in [1]. Here we show a possible approach to remove the main limitation that we put in [2,3], namely the hypothesis of binary ruptures. We notice that, since the scattering operator involves a breakage event, it must be modified accordingly in order to allow volume scattering with *multiple exit*.

Indeed multiple breakage has been considered by many Authors and many other references therein) but with the philosophy of capturing a global information about breakage, in view of the difficulty of analyzing the single modes. The aim we are pursuing here is instead to emphasize the contribution of each breakage class to the rate of change of the distribution function. The model is more complicate if compared with previous approaches because the probability functions that describe each rupture mode appear explicitly. However in such a way it becomes possible, at least in principle, to investigate the contribution to f due to each mode *individually*. The analysis we present is complete: we prove the

well posedness of the Cauchy problem under rather general hypotheses. We allow a parent droplet to break in N pieces where N can be any finite positive integer greater or equal than two. The breakage frequency α_k of the $k-$th mode is allowed to tend to infinity as v tends to v_{m} (as in [1]).

References

[1] Borsi, I. *Dynamics of liquid–liquid dispersions with unbounded fragmentation kernel*, Adv. Math. Sci. Appl., **11** (2001), no. 2 571–591.

[2] Fasano, A. and Rosso, F. *Analysis of the Dynamics of Liquid–Liquid Dispersions*, Progress in Industrial Mathematics at ECMI 98, (Stuttgart Leipzig), (L. Arkeryd, J. Bergh, P. Brenner, and Pettersson eds.), Teubner, 1999, 214–221.

[3] Fasano, A. and Rosso, F., *A new model for the dynamics of dispersions in a batch reactor*, Proceedings of the Symposium Organized by the Sonderforschungsbereich 438 on the Occasion of Karl-Heinz Hoffman's 60th Birthday, (Berlin) (H. J. Bungartz and R. H. W. Hoppe and Ch. Zenger eds.), Lectures in Applied Mathematics, Berlin Springer Verlag, (2000) 123–141.

The Entropy Principle: from Continuum Mechanics to Hyperbolic Systems of Balance Laws

Tommaso Ruggeri

We discuss the different roles of the entropy principle in modern thermodynamics. We start with the approach of rational thermodynamics in which the entropy principle becomes a selection rule for physical constitutive equations. Then we discuss the entropy principle for selecting admissible discontinuous weak solutions and to symmetrize general systems of hyperbolic balance laws. A particular attention is given on the local and global well-posedness of the relative Cauchy problem for smooth solutions. At the end we give some recent results on closure procedure for the moments theory associated to the Boltzmann equation (Extended Thermodynamics).

Some Two-Phase Problems in Fluid Mechanics

Rodolfo Salvi

Multiphase problems in fluid mechanics are very interesting phenomena which are often found in the nature and engineering applications. In this paper a unified framework for two-phase systems in fluid mechanics is developed. Assuming as order parameter the characteristic function of the region occupied with one phase, we pose the basic equations in purely Eulerian description for fluid/solid, fluid/rigid body (self-propelled type problems) systems. Then we develop an existence theory for the systems fluid/viscoelastic solid and fluid/selfpropelled rigid body.

Classical Solvability of the Coupled System Modelling a Heat-Convergent Poiseuille-Type Flow

Timofey N. Shilkin

We consider the coupled system of two nonlinear scalar parabolic equations modelling a simple uni-directional Poiseuille-type flow of a homogeneous incompressible Newtonian fluid whose viscosity is a temperature-dependent function. The energy balance equation of this system takes into account the phenomena of the viscous energy dissipation. We prove existence of a classical solution to this system on an arbitrary interval of time. The smooth solution turns out to be unique in a wider class of weak solutions.

Optimal Urban Networks and Pricing Policies

Eugene Stepanov

Various models for constructing an optimal urban transportation network in a city with given densities of population and services are discussed. The models are based on the Monge-Kantorovich theory of optimal transportation and can be viewed as free boundary problems for transport equations. The same models stem also from some classical shape optimization problems. Existence, qualitative topological and geometrical properties as well as regularity of optimal networks is studied. Models for the optimal choice of the pricing policy for the use of such networks are also discussed.

Local Solvability for Systems of Non-Principal Type

Sergio Spagnolo

The purpose of this lecture is to show that some techniques of matrix algebra and microlocal analysis, recently introduced in the study of Cauchy problems for weakly hyperbolic equations, can be adapted to investigate also the *local surjectivity* of certain types of linear equations or systems of non-principal type.

In particular we consider a special class of systems of the form

$$D_t u \ + \ \sum_{j=1}^{n} A_j(t,x) D_{x_j} u \ + \ A_0(t,x) u \ = \ f(t,\, x), \qquad (*)$$

which includes in particular all the hyperbolic systems, which result to be locally solvable in suitable Gevrey classes. The main assumption is that the characteristic matrix of (*), i.e., the matrix $\sum A_h(t,x)\xi_h$, has eigenvalues which satisfy the dicothomy:

$$\text{either} \quad \Im\lambda_j(t,x,\xi) \geq 0 \quad \forall j,t,x, \qquad \text{or} \quad \Im\lambda_j(t,x,\xi) \leq 0 \quad \forall j,t,x,$$

for each fixed $\xi = (\xi_1, \ldots .\xi_n) \in \mathbf{R}^n$.

Smoothness of Solutions of PDE's

David S. Tartakoff

I will present an overview of the subject of (local) smoothness of solutions ("hypoellipticit") of partial differential equations and then discuss some recent work on real analytic and Gevrey regularity of solutions of some interesting equations arising from various branches of mathematics.

Regularity for Parabolic Free Boundary Problems

Nina N. Uraltseva

The progress in the study of the regularity of solutions to non-linear problems for PDEs achieved on the last two decades is mainly based on the blow-up technique and the various monotonicity formulas. This approach has being successfully applied in the study of free boundary problems. In my talk the review of recent results obtained on this way for parabolic free boundary problems will be given

Transmission Problems with Highly Conductive Fractal Layers

Maria Agostina Vivaldi

The aim of this talk is to describe second order transmission problems involving a layer of fractal type, which is imbedded in an Euclidean domain. Boundary value problems, in which boundaries are "large" and volumes are "small" emerge naturally in transmission problems of absorption or irrigation type, where surface effects are enhanced. In this context, fractal boundaries and fractal layers provide new interesting setting. Fractals are geometric objects with highly non-Euclidean characteristics: despite their tricky geometry, there are however large families of fractals which possess a very rich analytic structure. So we are able to study fractals both as intrinsic bodies, in which is possible to give a suitable notion of Laplacean and as boundaries of Euclidean domains supporting traces of functions belonging to classic spaces like mains supporting traces of functions belonging to classic spaces like Sobolev spaces. Or possibly as bodies and boundaries at the same time, when they occur as highly conductive layers inside a Euclidean domain, which is the situation we focus on in this presentation. We are particulary interested in the case of highly conductive layers: these layers enjoy higher conductivity with respect to the surrounding space and, because of that, they absorb energy and convey intrinsic diffusion. From the analytical point of view one deals with transmission conditions involving at the same time traces of functions from classic Sobolev spaces and intrinsic Laplacians within the layer.